3.1. Domain Theoretical Calculation (Single Crystal)

Consistent with domain theory, the volume integral in the energy expression for the anisotropic ferromagnet will be ignored. Instead, the total energy sufficient for predicting magnetic equilibrium states will be written.

$$\mathcal{E} = \mathcal{E}_{ex} + \mathcal{E}_A + \mathcal{E}_d + \mathcal{E}_H$$

where each term refers to an energy density. The first term is the exchange energy while the second is the crystalline and magnetoelastic anisotropy energy. $\mathcal{E}_d = -\frac{1}{2}\vec{H}_d \cdot \vec{M}$ is the demagnetizing energy. $\mathcal{E}_H = -\vec{H}_e \cdot \vec{M}$ is the additional term included by the Legendre transformation and is just the interaction energy of the ferromagnet with the external magnetic field. It will be necessary to obtain each term for the problem of interest.

The total anisotropy energy from conventional magnetoelastic theory is

$$\mathcal{E}_A = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + b_1(\alpha_1^2 e_{11} + \alpha_2^2 e_{22} + \alpha_3^2 e_{33})$$

$$+ 2b_2(\alpha_1\alpha_2 e_{12} + \alpha_2\alpha_3 e_{23} + \alpha_3\alpha_1 e_{31}).$$

Uniaxial strain along a line colinear with the unit vector $\vec{n}$ can be written in the tensor form

$$e_{ij} = e n_i n_j$$

where $e = (\rho_o/\rho) - 1$ is the extension[27] along the direction of uniaxial strain. $\vec{n}$ is arbitrarily oriented with respect to the crystal axis. In the present work, interest lies in shock induced anisotropy. In shock wave studies, strains in the large elastic and plastic regions are obtained. Thus, for many magnetic materials, the crystalline anisotropy energy is 10 to 30 times smaller than the induced anisotropy energy in this strain region.

Therefore, the crystalline anisotropy energy will be ignored. The anisotropy energy of interest becomes

$$\mathcal{E}_{me} = b_1 e(\alpha_1^2 n_1^2 + \alpha_2^2 n_2^2 + \alpha_3^2 n_3^2) + 2b_2 e(\alpha_1\alpha_2 n_1 n_2 + \alpha_2\alpha_3 n_2 n_3 + \alpha_3\alpha_1 n_3 n_1). \quad (3.1)$$

To proceed with the domain theory analysis of the shock induced anisotropy effect, two single crystal problems will be treated concurrently. These will be called the <100> problem and the <111> problem. The <100> problem corresponds to a state of uniaxial strain along a <100> axis with a perpendicular applied field. The <111> problem corresponds to a state of uniaxial strain along a <111> axis with a perpendicular applied field. These two fundamental problems have their analogs in the thermodynamic inverse of this effect. They are magnetostriction along the <100> and <111> axes.[4] The magnetoelastic constants, b_1 and b_2, will be found to relate in a similar way to the magnetostriction constants, λ_{100} and λ_{111}.

In the spirit of domain theory, models for the domain structure must be postulated. Energies corresponding to each model are then obtained and compared. From this, conclusions are drawn as to the most probable domain structure. Figure 3.2 shows the domain structure models which will be considered. Domain walls normal to the strain axis are not expected. This is because the variation in the magnetization direction through the domain wall cannot be made without allowing $\vec{\nabla}\cdot\vec{M}$ to deviate from zero. $\vec{\nabla}\cdot\vec{M} \neq 0$ in the domain wall implies magnetic volume poles in the wall and, hence, a high demagnetizing energy. This would be energetically unfavorable. Domains of closure are not expected due to the high induced anisotropy energy.